

Littlest Higgs Model and Unitarity Constraints

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The Littlest Higgs Model is constrained using the tree-level perturbative arguments, similar to those employed to obtain the Higgs mass bound. It is found that the perturbative unitarity violations set in at energy scales $\sim 1.5\chi f$, where f is the analog of pion decay constant and sets the scale for the masses in the theory and also the strong interaction scale and χ is a dimensionless parameter which together with f sets the mass scale of heavy scalars in the theory.

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I. INTRODUCTION

The Standard Model (SM) of particle physics seems to agree with almost all the experimental data and has been tremendously successful in predicting particle properties and interactions. However, till date the mechanism of electroweak symmetry breaking is a mystery. The most popular one, the Higgs mechanism, requires fundamental scalar(s) in the theory, though none has been observed till date (for a review of the basic ideas involved in Higgs mechanism, some of the un-answered questions and remedies, see [1]). Indirect evidences point to a light Higgs boson [2]. A fundamental scalar in the theory implies that the mass term for this scalar particle will receive large quadratic corrections proportional to the largest scale, which generically would be identified with the Planck scale or some other very high fundamental scale in the full theory, including gravity finally. This leads to a large disparity, called the *hierarchy problem*, between the electroweak scale, fixed by the Fermi constant, and this high scale. To circumvent this problem, many solutions have been proposed and all of them have invited intense activity. Supersymmetry (for an introduction of basic ideas see [3]) is the most attractive of these proposals. The symmetry between the fermions and bosons renders the quadratic corrections to the Higgs mass contribution small provided the mass splitting between the SM particles and their super-partners is not too large, further implying that the supersymmetry breaking scale is not too large compared to the electroweak scale. However, there is no compelling reason to believe the existence of low energy supersymmetry. The other possibility which has attracted a lot of attention in the last couple of years is the idea that the four dimensional Planck scale is just a derived quantity while the fundamental scale of gravity can be as low as TeV [4]. This idea required the existence of one or more, compact or non-compact, spatial extra dimensions.

Almost all the alternative scenarios invoked in order to circumvent the hierarchy problem have their own set of drawbacks or short comings. In particular, the fact that they require the introduction/inclusion of a large number of new, and sometimes very exotic, particles makes them very speculative and in some sense reduce the predictive power of the theory as a whole.

Given the state of affairs, it seems very reasonable to look for other possible ways of getting across this problem. Recently, there has been a revived interest in the idea of the Higgs boson being a Pseudo-Goldstone Boson (PGB) [5]. Exploiting this idea, Arkani-Hamed et al [6] have proposed a minimal model, the Littlest Higgs model, where there are no elementary scalars in the theory, thus avoiding the issue of quadratic divergence right from the start. Motivated by the dimensional deconstruction ideas [7], it was shown that the little Higgs models can be successfully realised [8] with the basic feature that the quadratic divergences to the Higgs mass are absent to one loop level. In the little Higgs models, very similar to models with dynamical symmetry breaking (for details see for example [9]), there are no elementary scalars. An attractive and perhaps plausible way to obtain these scalars is to imagine a scenario where some strong dynamics is responsible for binding the elementary fermions into composite scalars, which subsequently play the role of the Higgs boson. At the scales relevant for such strong dynamics, the fermions condensate and break the “strong” group, thereby giving rise to Goldstone bosons, the details and number of which are model dependent.

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The situation is very similar to chiral symmetry breaking in QCD [10]. However, it should be kept in mind that this might just be one of the possible ways of achieving the goal and that other possibilities are not ruled out apriori.

In the model considered in [6], it is assumed that the theory has an $SU(5)$ global symmetry which is spontaneously broken to $SO(5)$ yielding a large number of Goldstone bosons. This is assumed to happen at some large scale where the fermions form condensates and break the symmetry. The model assumes that there are two copies of $SU(2) \times U(1)$ (direct product groups) embedded in $SU(5)$ which break to $[SU(2) \times U(1)]_{SM}$ at the same scale as that of the global symmetry breaking. The presence of a larger symmetry group ensures that the Higgs mass is approximately protected and therefore avoids the problem of large quadratic divergences upto one loop level. The electroweak symmetry breaking is triggered via the Coleman-Weinberg mechanism.

Several variations of this idea have also been studied [11]. Some of the phenomenological aspects of such scenarios have also been explored [12]. In this note we constrain the minimal model [6] by invoking arguments based on tree level perturbative unitarity of the the scattering amplitudes.

II. LITTLEST HIGGS MODEL: A QUICK LOOK

Consider the littlest Higgs model [6] based on non-linear sigma model describing $SU(5)/SO(5)$ symmetry breaking, originating due to vacuum expectation value (VEV) $\sim \mathcal{O}(f)$ of a symmetric tensor. By dimensional analysis [13], this implies that the symmetry breaking scale $\Lambda \sim 4\pi f$. Let us further choose a convenient basis in which the VEV points in the following direction, labelled by Σ_0

$$\Sigma_0 = \begin{pmatrix} & & 1_{2 \times 2} \\ & 1 & \\ 1_{2 \times 2} & & \end{pmatrix} \quad (2.1)$$

Further, if X^a are the broken generators, then the Goldstone bosons (π^a) can be parameterized by the non-linear sigma field as

$$\Sigma(x) = e^{2i\Pi^T/f\Sigma_0} \quad (2.2)$$

where $\Pi = \pi^a X^a$. The breaking $SU(5) \rightarrow SO(5)$ yields fourteen Goldstone bosons out of which four are absorbed by the gauge fields (the heavy gauge fields in the model) to acquire masses when the $[SU(2) \times U(1)]^2$ sub-group is gauged to $SU(2)_L \times U(1)_Y$. In terms of the remaining ten massless scalars, the Goldstone boson matrix Π can be written as

$$\Pi = \begin{pmatrix} 0 & \frac{h^\dagger}{\sqrt{2}} & \phi^\dagger \\ \frac{h}{\sqrt{2}} & 0 & \frac{h^*}{\sqrt{2}} \\ \phi & \frac{h^T}{\sqrt{2}} & 0 \end{pmatrix} \quad (2.3)$$

where h is a complex doublet under the SM gauge group while ϕ is a complex triplet forming a 2×2 symmetric tensor. The breaking of $[SU(2) \times U(1)]^2$ to the SM gauge group means that the four vector bosons acquire masses of the order of f and therefore are heavy. The non-linear sigma model Lagrangian (to the leading order) reads

$$\mathcal{L}_\Sigma \sim \frac{f^2}{4} \text{Tr} |\mathcal{D}_\mu \Sigma|^2 \quad (2.4)$$

where \mathcal{D}_μ is the relevant covariant derivative including the gauge fields and couplings corresponding to both the copies of $SU(2) \times U(1)$:

$$\mathcal{D}_\mu \Sigma = \partial_\mu \Sigma - i \sum_{j=1}^2 [g_j W_j^a (Q_j^a \Sigma + \Sigma Q_j^{aT}) + g'_j B_j (Y_j \Sigma + \Sigma Y_j)] \quad (2.5)$$

where g_j , g'_j are the gauge couplings, W_j , B_j are the gauge bosons and Q_j^a and Y_j are the generators of gauge transformations. The gauge couplings break the global symmetry explicitly. However, no single interaction breaks the complete symmetry and therefore the Higgs mass is partially protected. Expanding Σ about Σ_0 gives the linearized theory and breaks the local gauge symmetry to the SM gauge symmetry. The massless gauge bosons are identified

with the SM gauge bosons. In what follows, we closely follow the notations and conventions chosen in Han et al. [12]. The breaking of the local gauge symmetry to the SM gauge group generates masses for four gauge bosons. Linear combinations of W_j and B_j are respectively constructed such that one set becomes massive while the other set remains massless and is identified with the SM gauge bosons. Also, in terms of the gauge couplings g_j and g'_j the mixing angles are defined as:

$$s = \frac{g_2}{(g_1^2 + g_2^2)^{\frac{1}{2}}}, \quad s' = \frac{g'_2}{(g_1'^2 + g_2'^2)^{\frac{1}{2}}} \quad (2.6)$$

The SM gauge couplings are finally identified as

$$g = g_1 s = g_2 c \quad g' = g'_1 s' = g'_2 c' \quad (2.7)$$

Let v and v' denote the Higgs and triplet VEVs respectively. Expanding the Lagrangian in terms of the fields h and ϕ , and minimising the potential, we arrive at an expression for the triplet mass (m_Φ). By demanding that the triplet mass squared is a positive definite quantity, the two VEVs get related in the following fashion:

$$\frac{v'^2}{v^2} \leq \frac{v^2}{16f^2} \quad (2.8)$$

Therefore, we can now trade off v' for v and f .

In the fermionic sector, one of the main concerns is to generate a large top-Yukawa coupling and at the same time ensuring that there are no large quadratic corrections due to top quark. In the present scenario, this is accomplished by introducing a pair of coloured Weyl fermions labelled - \tilde{t} and \tilde{t}^c in addition to the third family doublet comprising of bottom and top quark. Expanding the relevant piece to first order in h and rearranging by suitably combining the different fields yields large mass $\sim \mathcal{O}(f)$ for the vector like fermion and generates the desired top-Yukawa coupling. In this minimal framework, no additional fields are introduced for any of the other fermions.

Finally, the electroweak symmetry breaking is induced via the Coleman-Weinberg mechanism [14] and therefore the quadratic corrections to Higgs mass are absent to one loop level. Therefore, below the scale f , this minimal model has the particle content that is identical to SM and nothing else. Moreover, as the scale f is approached, there is a minimal set of heavy fields that gets introduced. This is in some sense a nice economical feature of the model. Therefore, in this scenario, at least two operators are needed to completely break the global symmetry that protects the Higgs mass, forbidding quadratic divergences at one loop level. The Higgs mass gets quadratically corrected at the two loop level and hence is much smaller than Λ . We thus have a situation : $\Lambda > f > v$ such that the successive hierarchies are small. Above the global symmetry breaking scale, we require a UV completion mechanism for the theory. However, for the present study, we don't need to be bothered about the details of such a mechanism.

III. UNITARITY CONSTRAINTS

The question generally asked while relying on perturbative calculations is regarding the energy scales upto which the theory can be treated perturbatively. Put in another form, it is desirable to have an estimate of the cut-off scale governing the low energy dynamics in the perturbative fashion. It is well known that such an issue can be addressed to a fair degree of accuracy on the arguments based on tree level perturbative requirements of the scattering amplitudes. In SM, it is known that the Higgs boson plays a crucial role in restoring the unitarity of longitudinal gauge boson scattering amplitudes and this in turn gives a bound on the Higgs mass [15].

We use these very ideas to get an idea of the scale upto which we can naively treat little Higgs models perturbatively, without bothering about the strong dynamics setting in. To this effect, we study scattering of longitudinal vector bosons in this model. We expect that the longitudinal gauge boson interactions should bring out any features of the strong interactions because the *equivalence theorem* [16] in its simplest and naive form states that for energies much larger than the vector boson masses, the scattering of the longitudinal gauge bosons can be viewed as scattering of the Goldstone bosons. This simplifies the calculations drastically. In practice, the polarization vectors for the gauge bosons are taken to be

$$\epsilon^\mu(p_V) = \frac{p_V^\mu}{M_V} + \mathcal{O}(M_V^2/E^2)$$

where p_V and M_V are the vector boson momentum and mass respectively and $E \gg M_V$ characterises the typical energy scale involved in the scattering process. It may be argued that this naive form of the equivalence theorem is not applicable when dealing with effective theories or chiral Lagrangians because of the fact that effective theories imply the low energy limit of the full theory while equivalence theorem implies that we have to take the high energy limit. These two limits have to be simultaneously taken and it may not always be possible to successfully implement this procedure. Also, in the case of effective theories, since the energy scales are restricted to a certain value, there is an infinite series of higher dimensional operators in the low energy theory and in such a situation the validity of the naive form of equivalence theorem is in question. These aspects have been discussed in [17]. Also, we are interested in the longitudinal gauge boson scattering amplitudes. But under a general Lorentz transformation, the longitudinal and transverse components of the gauge field can mix and therefore the replacement of the longitudinal components by their associated Goldstone bosons can lead to Lorentz non invariant results [18]. However, the requirement, $E \gg M_V$ circumvents such difficulties and defines the suitable Lorentz frame(s). Also, this ensures that the $\mathcal{O}(M_V^2/E^2)$ terms are actually small and can be safely neglected. Furthermore, while considering the effective theory amplitudes, the correct energy regime to be employed to use the equivalence theorem in its simplest form and without bothering about other corrections (except for the renormalization factors arising at loop level), is $M_V \ll E < 4\pi\epsilon$ where ϵ is the energy scale that fixes the gauge boson masses [19].

In the present case, the model predicts a very light Higgs, whose mass is protected by some approximate symmetries upto one loop level and a complex triplet of scalars. Expanding the sigma model Lagrangian to relevant order, it is clear that there are additional corrections to the usual SM vertices. Also, the scattering processes will receive additional contributions from the new vertices and particles present. In the SM, the Higgs boson restores the unitarity of the amplitudes and there is a very delicate cancellation between various diagrams that effects such a beautiful restoration. However, in the case when there are deviations, however small, from the exact SM vertices and more so when there are additional particles in the theory, such a cancellation may not be operative so effectively. Following the expanded Lagrangian as in Han et al [12], it is easy to convince that the pure SM pieces will be nicely behaved. All the bad behaviour contributions either stem out due to the presence of the triplet of scalars or some residual contribution from the corrections to SM vertices. In the notation of Han et al [12], we make the following allowed phenomenological choices,

$$c = s \quad c' = s' \quad \frac{v'}{v} = \frac{v}{\chi f} \quad (3.1)$$

with $\chi > 4$ such that Eq.(2.8) is satisfied. We choose different values of the parameter χ and investigate the effects on unitarity violation.

We consider the coupled longitudinal vector boson scattering amplitudes (keeping in mind the discussion above regarding the validity and use of equivalence theorem in chiral/effective theories) which on using these choices result in the following $J = 0$ partial wave amplitudes:

$$\begin{aligned} \mathcal{M}(W_L W_L \rightarrow W_L W_L) = & \left(-\frac{ig^2}{64\pi M_W^2} \right) \left[f_{WWH}^2 \left(2m_H^2 + \frac{m_H^4}{s - m_H^2} - \frac{m_H^4}{s} \ln(1 + s/m_H^2) \right) \right. \\ & + f_{WW\Phi^0}^2 \left(2m_\Phi^2 + \frac{m_\Phi^4}{s - m_\Phi^2 + im_\Phi \Gamma_\Phi} - \frac{m_\Phi^4}{s} \ln(1 + s/m_\Phi^2) \right) \\ & \left. + f_{WW\Phi^{--}}^2 \left(m_\Phi^2 - \frac{s}{2} - \frac{m_\Phi^4}{s} \ln(1 + s/m_\Phi^2) \right) \right] \end{aligned} \quad (3.2)$$

$$\begin{aligned} \mathcal{M}(Z_L Z_L \rightarrow Z_L Z_L) = & \left(-\frac{ig^2}{64\pi M_W^2} \right) \left[f_{ZZH}^2 \left(3m_H^2 + \frac{m_H^4}{s - m_H^2} - 2\frac{m_H^4}{s} \ln(1 + s/m_H^2) \right) \right. \\ & \left. + f_{ZZ\Phi^0}^2 \left(3m_\Phi^2 + \frac{m_\Phi^4}{s - m_\Phi^2 + im_\Phi \Gamma_\Phi} - 2\frac{m_\Phi^4}{s} \ln(1 + s/m_\Phi^2) \right) \right] \end{aligned} \quad (3.3)$$

$$\mathcal{M}(W_L W_L \rightarrow Z_L Z_L) = \left(-\frac{ig^2}{64\pi M_W^2} \right) \left[\left(m_H^2 + \frac{m_H^4}{s - m_H^2} \right) + (f_{WWH} - 1)(f_{ZZH} - 1) \frac{s^2}{s - m_H^2} \right]$$

$$\begin{aligned}
& + f_{WW\Phi^0} f_{ZZ\Phi^0} \frac{s^2}{s - m_\Phi^2 + im_\Phi \Gamma_\Phi} \\
& + f_{WZ\Phi^-}^2 \left(2m_\Phi^2 - s - 2 \frac{m_\Phi^4}{s} \ln(1 + s/m_\Phi^2) \right) \Big]
\end{aligned} \tag{3.4}$$

where

$$\begin{aligned}
f_{WWH} &= 1 - \frac{v^2}{3f^2} - \frac{s_0^2}{2} - 2\sqrt{2}s_0 \frac{v'}{v} & f_{ZZH} &= 1 - \frac{v^2}{3f^2} - \frac{s_0^2}{2} + 4\sqrt{2}s_0 \frac{v'}{v} \\
f_{WW\Phi^0} &= s_0 - 2\sqrt{2} \frac{v'}{v} & f_{WW\Phi^{--}} &= 4 \frac{v'}{v} \\
f_{ZZ\Phi^0} &= s_0 - 4\sqrt{2} \frac{v'}{v} & f_{WZ\Phi^-} &= 2 \frac{v'}{v}
\end{aligned} \tag{3.5}$$

with

$$s_0 \simeq 2\sqrt{2} \frac{v'}{v} \quad m_\Phi = \frac{\sqrt{2}m_H f}{v} \frac{1}{[1 - (4v'f/v^2)^2]^{\frac{1}{2}}} \tag{3.6}$$

where the decay width of the triplet, Γ_Φ has been explicitly included in the above amplitudes.

In writing $\mathcal{M}(W_L W_L \rightarrow Z_L Z_L)$, we've deliberately separated the pure SM (the first two terms in the parenthesis) and the new contributions. It is clear that $f_{WWH}, f_{ZZH} \rightarrow 1$ while $f_{WW\Phi^0}, f_{ZZ\Phi^0}, f_{WW\Phi^{--}}, f_{WZ\Phi^-} \rightarrow 0$, when $v/f \rightarrow 0$ and the $J=0$ amplitudes correspondingly reduce to the pure SM amplitudes. Also to be noted is the fact that with these choices of the parameters Eq.(3.1), there are no contributions to WW scattering amplitude due to heavy gauge bosons.

The coupled channels form a 2×2 matrix. This matrix is diagonalised and the unitarity constraint is imposed on the larger of the two eigenvalues ie if λ denotes the larger eigenvalue then the unitarity constraint reads

$$|Re(\lambda)| \leq \frac{1}{2} \tag{3.7}$$

giving an inequality in terms of $m_H, \sqrt{s}, f, \Gamma_\Phi$ and the parameter χ . For this study, we choose $m_H = 115$ GeV as indicated by the LEP results [20]. This considerably simplifies our discussion as well because the pure SM amplitudes are well behaved at high energies and respect the unitarity conditions for a light Higgs.

Furthermore, in the notation of [12] and as indicated in Eq.(3.6), s_0 is very close to $2\sqrt{2}v'/v$ and therefore in such a situation $f_{WW\Phi^0} \rightarrow 0$. To investigate the implications of the unitarity condition, we solve Eq.(3.7) for different choices of the parameters. The results are shown in Table 1.

For a light Higgs as we are considering, the SM contributions asymptotically approach a constant value and therefore

$\chi \rightarrow$ f (TeV) ↓	5	10	25
0.5	4	8.5	14.5
1	8.2	16.5	44
2	16.5	32.2	83.2

TABLE I: Unitarity violation scale (in TeV) for various values of f and χ

do not bother us with any significant effects. Also to be noted is that in the regime $s \gg m_\Phi^2$ ie when all terms of the form m_Φ^2/s and higher powers of the same are safely neglected, the amplitudes scale as $s/(f^2\chi^2)$ which is clearly depicted in Table 1 as well. In fact, it is not hard to conclude that the scale of unitarity violation sets in at

$$E_{unitarity} = \sqrt{s}_{critical} \sim (1.5 - 1.7) \chi f \tag{3.8}$$

Further, to ensure that the results are not faked by any resonant behaviour, we have chosen $\Gamma_\Phi = 50\text{GeV}$. This may seem a bit artificial, but for the present study, it serves the purpose of keeping the amplitudes well below the unitarity limits even when the center of mass energy is close to the triplet mass. Making Γ_Φ smaller will only result in a very sharp and narrow peak.

For comparison, we list the triplet mass for various χ values:

$$\begin{aligned} m_\Phi(\chi = 4) &= \infty & m_\Phi(\chi = 5) &\sim 1.1f \\ m_\Phi(\chi = 10) &\sim 0.71f & m_\Phi(\chi = 25) &\sim 0.66f \end{aligned}$$

Clearly, increasing χ has the effect of decreasing the triplet mass and as can be seen from Table 1, this has the effect of making the scale of unitarity violation larger, as expected. Letting χ take a large value results in a much larger scale at which the unitarity violations show up. However, this variation of the triplet mass with χ is not found for very large values of χ for which the triplet mass becomes practically independent of the χ value, as can be seen from Eq.(3.6) and using the relation between v and v' . From the present analysis, it is quite transparent that $\sqrt{s}_{critical} > 2f$ for all the allowed values. This has the simple interpretation that, modulo multiplicative factors of order unity, the theory under consideration respects perturbative unitarity upto these scales. In particular, we can safely conclude that the effective theory description is valid upto scales $\mathcal{O}(2m_\Phi)$ and in fact, much beyond. This is in agreement with the general results and arguments advocated in [21] for the validity and reliability of effective theory descriptions.

These limits are not changed significantly if we vary the Higgs mass even upto 150 GeV. For $\chi = 5$, Eq.(3.6) implies that the mass of the heavy scalars (triplet) $\sim 1.1f$ and mass of the heavy top $\sim 1.5f$. Also, the masses of the heavy $SU(2)$ gauge bosons are of the order of the triplet mass. The heavy $U(1)$ gauge boson is not really very heavy, $\geq 0.15f$, and can cause observable effects in the precision measurements. Park and Song [12] have pointed out that even a 200 GeV heavy $U(1)$ gauge boson gives a negligible contribution to muon magnetic moment.

In this study we have been concerned with the unitarity of the lighter of the two sets of vector bosons predicted by the littlest Higgs model. The unitarity limits can change if the heavier set scattering amplitudes are also accounted for. Nonetheless, the limits obtained from the lighter gauge boson scattering amplitudes are going to be much more stringent and we therefore stick to them in the present note.

IV. CONCLUSIONS

In this note, we have studied the tree-level perturbative unitarity constraints on the littlest Higgs model. It is found that demanding the perturbative validity of the SM, in the minimally modified form, we obtain the unitarity violating scale $E_{unitarity} \sim 1.5\chi f$ with $\chi > 4$ always. For the limiting case, when χ is marginally greater than 4, $E_{unitarity} \sim (6-7)f$, smaller than $4\pi f$, the scale of strong dynamics. For larger values of χ , the requirement that $M_V \ll E \ll 4\pi f$ is not met and therefore, the equivalence theorem cannot be used in such a simple manner. To avoid such difficulties and to get physically meaningful results, it is therefore judicious to have $\chi \sim 5$ so as to satisfy the above requirement. In this study, we have only considered the longitudinal gauge boson scattering. Perhaps, the limits obtained using the present analysis can be pushed a bit if we consider other channels involving the Higgs boson and top quark as well. Going beyond the tree-level would introduce renormalization corrections to the equivalence theorem and can further modify the present limits.

The limits on f obtained in the present note agree with the general expectation based on other phenomenological explorations [12]. However, one important point that can cause some modification in these values is the fact that there is some amount of hidden uncertainty pertaining to the specific UV completion mechanism for such a model. Nevertheless, we believe that such an effect will only introduce a multiplicative correction factor $\mathcal{O}(1)$. Also, we have chosen very specific values for the various mixing angles in our study. Changing these values does not significantly affect our results as the corrections introduced due to such changes will be small. Only detailed studies can make very specific and accurate predictions about these mixing parameters which can, in principle, make significant effects in some other sectors.

In conclusion, we like to mention that with the chosen parameter set, the unitarity bound on the parameter f which sets the mass scale for the heavy particles and also the scale of the strong dynamics, is in gross agreement with the phenomenological constraints. Similar result can be expected for any other variant of this minimal model considered here.

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